NASA TECHNICAL MEMORANDUM

NASA TM X-71986 COPY NO.

AN ITERATIVE TECHNIQUE TO STABILIZE A LINEAR TIME INVARIANT

MULTIVARIABLE SYSTEM WITH OUTPUT FEEDBACK

VISWANATHAN SANKARAN
NRC POSTDOCTORAL RESEARCH ASSOCIATE
FLIGHT DYNAMICS AND CONTROL DIVISION

JULY 1, 1974

(NASA-TM-X-71986) AN ITERATIVE TECHNIQUE TO STABILIZE A LINEAR TIME INVARIANT MULTIVARIABLE SYSTEM WITH OUTPUT FEEDBACK (NASA) 12 p HC \$4.00 CSCL 09C



This informal documentation medium is used to provide accelerated or special release of technical information to selected users. The contents may not meet NASA formal editing and publication standards, may be revised, or may be incorporated in another publication.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION LANGLEY RESEARCH CENTER, HAMPTON, VIRGINIA 23665

TM X-71986 4. Title and Subtine An Iterative Technique to Stabilize a Linear Time Invariant Multivariable System with Output Feedback 7. Authoris Dr. Viswanathan Sankaran INC Fostdoctoral Research Associate, FDCD 9. Performing Organization Names and Addres NASA Langley Research Center Hampton, VA 23665 12. Spowdering Agency Name and Addres National Aeronautics and Space Administration Washington, IC 20546 15. Supplementary Notes 16. Abstract The problem of finding a physically realizable, i.e. constant gain, output-feedback controller which will stabilize an unstable plant is one of the oldest and most fundamental problems in control theory. In spite of its long history, this problem still remains unsolved even in the seemingly simple case of linear plants. In this paper, an iterative procedure for determining the constant gain matrix that will stabilize a linear constant multivariable system using output feedback is described. The use of this procedure avoids the transformation of states of the plant, we are able to give a rather general solution. In the case in which the states exceed the product of input and output vector dimensions is greater than the number of states of the plant, we are able to give a rather general solution. In the case in which the states exceed the product of input and output vector dimensions we are able to present a least square solution which may not be stable in all cases. The results are illustrated with examples. 17. Key Word (Supposed by Authoris) [STAR compony underlined] 18. Distribution Sustement 19. Security Clossif. (of this report) 20. Security Clossif. (of this page) 21. No. of Pages 22. Prior*						
An iterative Technique to Stabilize a Linear Time Invariant Multivariable System with Output Feedback 7. Author(s) Dr. Viswamathan Sankaran NNC Postdoctoral Research Associate, FDCD 9. Petroming Organization Name and Address NASA Iangley Research Center Hampton, VA 23665 12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, DC 20566 15. Supplementary Notes 16. Abstract The problem of finding a physically realizable, i.e. constant gain, output-feedback controller which will stabilize an unstable plant is one of the oldest and most fundamental problems in control theory. In spite of its long history, this problem still remains unsolved even in the seemingly simple case of linear plants. In this paper, an iterative procedure for determining the constant gain matrix that will stabilize a linear constant multivariable system using output feedback is described. The use of this procedure avoids the transformation of variables which is required in other procedures for the case in which the product of the output and input vector dimensions is greater than the number of states of the plant, we are able to give a rather general solution. In the case in which the states exceed the product of input and output vector dimensions we are able to present a least square solution which may not be stable in all cases. The results are illustrated with examples. 17. Key Word (Supputed by Author(s)) (STAR category underlined) 18. Distribution Sutement 19. No. of Pages 12. Price* 19. No. of Pages 12. Price* 19. No. of Pages 12. Price* 19. Security Closs! (of this report) 20. Security Closs! (of this page) 21. No. of Pages 12. Price* 22. Price* 23. Price* 24. Price* 25. Report Date of No. 26. Petroming Organization No. 28. Petroming Organization No. 28. Petroming Organization No. 29. P	1. Report No.	2. Government Accession	en No.	3. Recipiont's Catalog	No.	
An Iterative Technique to Stabilize a Linear Time Invariant Multivariable System with Output Peedback 7. Author(s) Dr. Viswanathan Sankaran INC Fostdoctoral Research Associate, FDCD 9. Performing Organization Names and Addres NASA Langley Research Center Hampton, VA 23665 12. Sponsoring Agency Name and Addres National Aeronautics and Space Administration Washington, DC 205% 15. Supplementary Notes 16. Abstract The problem of finding a physically realizable, i.e. constant gain, outputfeedback controller which will stabilize an unstable plant is one of the oldest and most fundamental problems in control theory. In spite of its long history, this problem still remains unsolved even in the seemingly simple case of linear plants: In this paper, an iterative procedure for determining the constant gain matrix that will stabilize a linear constant multivariable system using output feedback is described. The use of this procedure avoids the transformation of variables which is required in other procedures. For the case in which the product of the output and input vector dimensions is greater than the number of states of the plant, we are able to give a rather general solution. In the case in which the states exceed the product of find put and output vector dimensions we are able to present a least square solution which may not be stable in all cases. The results are illustrated with examples. 17. Key Words (Suppend by Author(s)) (STAR cotegory underlined) optimal control, output feedback Star Category 10 18. Security Clossit (of this report) 20. Security Clossit (of this report) 21. No. of Pape 22. Prior		·		5 Remort Data		
Invariant Multivariable System with Output Feedback 7. Author(s) Dr. Viswanathan Sankaran NRC Fostdoctoral Research Associate, FDCD 9. Performing Organization Name and Addres NASA Langley Research Center Hampton, VA 23665 12. Sponsoring Agency Name and Addres National Aeronautics and Space Administration Washington, DC 20546 15. Supplementary Notes 16. Abstract The problem of finding a physically realizable, i.e. constant gain, output-feedback controller which will stabilize an unstable plant is one of the oldest and most fundamental problems in control theory. In spite of its long history, this problem still remains unsolved even in the seemingly simple case of linear plants. In this paper, an iterative procedure for determining the constant gain autrix that will stabilize a linear constant multivariable system using output feedback is described. The use of this procedure avoids the transformation of variables which is required in other procedures. For the case in which the product of the output and input vector dimensions is greater than the number of states of the plant, we are able to give a rather general solution. In the case in which the states exceed the product of input and output vector dimensions we are able to present a least square solution which may not be stable in all cases. The results are illustrated with examples. 10. Note Unit No. 766-75-02-01 11. Context or Grant No. 12. Type of Report No. 13. Type of Report No. 14. Sponsoring Agency Code 15. Supplementary Notes 16. Abstract The problem of finding a physically realizable, i.e. constant gain, output-feedback solution. Supplementary Notes 16. Abstract The problem of finding a physically realizable, i.e. constant gain, output-feedback is described. The use of this procedure avoid the transformation of variables which is required in other procedures. For the case in which the product of the output vector dimensions is greater than the number of states of the plant, we are able to give a rather general solution. In the case in w						
7. Author(s) Dr. Viswanathan Sankaran NRC Fostdoctoral Research Associate, FDCD 9. Performing Organization Name and Address NASA Langley Research Center Hampton, VA 23665 10. Work Unit No. 766-75-02-01 11. Contract or Grant No. 12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, NC 20516 15. Supplementary Notes 16. Abstract The problem of finding a physically realizable, i.e. constant gain, output-feedback controller which will stabilize an unstable plant is one of the oldest and most fundamental problems in control theory. In spite of its long history, this problem still remains unsolved even in the seemingly simple case of linear plants. In this paper, an iterative procedure for determining the constant gain matrix that will stabilize a linear constant multivariable system using output feedback is described. The use of this procedure avoids the transformation of variables which is required in other procedures. For the case in which the product of the output and input vector dimensions is greater than the number of states of the plant, we are able to give a rather general solution. In the case in which the states exceed the product of input and output vector dimensions we are able to present a least square solution which may not be stable in all cases. The results are illustrated with examples. 17. Key Words (Supputed by Author(s)) (STAR catagory underlined) 18. Data/bution Statement Optimal control, output feedback Star Category 10 19. Security Otesit (of this report) 20. Security Closif (of this page) 21. No. of Press 22. Price*	<u>-</u>					
Dr. Viswanathan Sankaran MNC Fostdoctoral Research Associate, FDCD 10. Work Unit No. 766-75-02-01 NASA Langley Research Center Hampton, VA 23665 11. Contract or Grant No. 12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, DC 20546 16. Abstract The problem of finding a physically realizable, i.e. constant gain, output-feedback controller which will stabilize an unstable plant is one of the oldest and most fundamental problems in control theory. In spite of its long history, this problem still remains unsolved even in the seemingly simple case of linear plants. In this paper, an iterative procedure for determining the constant gain matrix that will stabilize a linear constant multivariable system using output feedback is described. The use of this procedures For the case in which the product of the output and input vector dimensions is greater than the number of states of the plant, we are able to give a rather general solution. In the case in which the states exceed the product of input and output vector dimensions we are able to present a least square solution which may not be stable in all cases. The results are illustrated with examples.	Invariant Multivariable System with Output Feedback			6. Performing Organia	ration Code	
9. Performing Organization Name and Address NASA Langley Research Center Hampton, VA 23665 12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, DC 20546 15. Supplementary Notes 16. Abstract The problem of finding a physically realizable, i.e. constant gain, output-feedback controller which will stabilize an unstable plant is one of the oldest and most fundamental problems in control theory. In spite of its long history, this problem still remains unsolved even in the seemingly simple case of linear plants. In this paper, an iterative procedure for determining the constant gain matrix that will stabilize a linear constant multivariable system using output feedback is described. The use of this procedure avoids the transformation of variables which is required in other procedures. For the case in which the product of the output and input vector dimensions is greater than the number of states of the plant, we are able to give a rather general solution. In the case in which the states exceed the product of input and output vector dimensions we are able to present a least square solution which may not be stable in all cases. The results are illustrated with examples. 17. Key Words (Supposted by Author(s)) (STAR cotspory underlined) optimal control, output feedback Star Category 10	7. Author(s)			8. Performing Organization Report No.		
9. Performing Organization Name and Address NASA Langley Research Center Hampton, VA 23665 12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, DC 20546 15. Supplementary Notes 16. Abstract The problem of finding a physically realizable, i.e. constant gain, output-feedback controller which will stabilize an unstable plant is one of the oldest and most fundamental problems in control theory. In spite of its long history, this problem still remains unsolved even in the seemingly simple case of linear plants. In this paper, an iterative procedure for determining the constant gain matrix that will stabilize a linear constant multivariable system using output feedback is described. The use of this procedure avoids the transformation of variables which is required in other procedures. For the case in which the product of the output and input vector dimensions is greater than the number of states of the plant, we are able to give a rather general solution. In the case in which the states exceed the product of input and output vector dimensions we are able to present a least square solution which may not be stable in all cases. The results are illustrated with examples. 17. Key Words (Supposted by Author(s)) (STAR cotspory underlined) optimal control, output feedback Star Category 10	Dr. Viswanathan Sankaran NRC Postdoctoral Research Associate, FDCD			10 Work Unit No		
NASA Langley Research Center Hampton, VA 23665 12. Sconsoring Agency Name and Address National Aeronautics and Space Administration Washington, DC 205%6 15. Supplementary Notes 16. Abstract The problem of finding a physically realizable, i.e. constant gain, output-feedback controller which will stabilize an unstable plant is one of the oldest and most fundamental problems in control theory. In spite of its long history, this problem still remains unsolved even in the seemingly simple case of linear plants. In this paper, an iterative procedure for determining the constant gain matrix that will stabilize a linear constant multivariable system using output feedback is described. The use of this procedures. For the case in which the product of the output and input vector dimensions is greater than the number of states of the plant, we are able to give a rather general solution. In the case in which the states exceed the product of input and output vector dimensions we are able to present a least square solution which may not be stable in all cases. The results are illustrated with examples. 17. Key Words (Suggested by Author(s)) (STAR catagory underlined) optimal control, output feedback Star Category 10						
Hampton, VA 23665 12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, DC 20546 15. Supplementary Notes 16. Abstract The problem of finding a physically realizable, i.e. constant gain, output-feedback controller which will stabilize an unstable plant is one of the oldest and most fundamental problems in control theory. In spite of its long history, this problem still remains unsolved even in the seemingly simple case of linear plants. In this paper, an iterative procedure for determining the constant gain matrix that will stabilize a linear constant multivariable system using output feedback is described. The use of this procedure avoids the transformation of variables which is required in other procedures. For the case in which the product of the output and input vector dimensions is greater than the number of states of the plant, we are able to give a rather general solution. In the case in which the states exceed the product of input and output vector dimensions we are able to present a least square solution which may not be stable in all cases. The results are illustrated with examples. 17. Key Words (Suggested by Author(s)) [STAR catagory underlined] 18. Distribution Statement 19. No. of Pages 22. Price*			-	<u> </u>		
National Aeronautics and Space Administration 14. Sponsoring Agency Code 15. Supplementary Notes 16. Abstract The problem of finding a physically realizable, i.e. constant gain, output-feedback controller which will stabilize an unstable plant is one of the oldest and most fundamental problems in control theory. In spite of its long history, this problem still remains unsolved even in the seemingly simple case of linear plants. In this paper, an iterative procedure for determining the constant gain matrix that will stabilize a linear constant multivariable system using output feedback is described. The use of this procedure avoids the transformation of variables which is required in other procedures. For the case in which the product of the output and input vector dimensions is greater than the number of states of the plant, we are able to give a rather general solution. In the case in which the states exceed the product of input and output vector dimensions we are able to present a least square solution which may not be stable in all cases. The results are illustrated with examples. 17. Key Words (Suggested by Author(s)) ISTAR category underlined) optimal control, output feedback Star Category 10 18. Distribution Statement 19. No. of Peges 22. Price*				11. Contract or Grant No.		
National Aeronautics and Space Administration Washington, DC 20565 16. Abstract The problem of finding a physically realizable, i.e. constant gain, output-feedback controller which will stabilize an unstable plant is one of the oldest and most fundamental problems in control theory. In spite of its long history, this problem still remains unsolved even in the seemingly simple case of linear plants. In this paper, an iterative procedure for determining the constant gain matrix that will stabilize a linear constant multivariable system using output feedback is described. The use of this procedure avoids the transformation of variables which is required in other procedures. For the case in which the product of the output and input vector dimensions is greater than the number of states of the plant, we are able to give a rather general solution. In the case in which the states exceed the product of input and output vector dimensions we are able to present a least square solution which may not be stable in all cases. The results are illustrated with examples. 12. Key Words (Suggested by Authoria) (STAR catagory underlined) optimal control, output feedback Star Category 10 18. Distribution Statement Optimal control, output feedback Star Category 10 20. Security Clocks, (of this page) 21. No. of Pages 22. Price*				13. Type of Report an	d Period Covered	
National Aeronautics and Space Administration Washington, DC 20546 16. Abstract The problem of finding a physically realizable, i.e. constant gain, output-feedback controller which will stabilize an unstable plant is one of the oldest and most fundamental problems in control theory. In spite of its long history, this problem still remains unsolved even in the seemingly simple case of linear plants. In this paper, an iterative procedure for determining the constant gain matrix that will stabilize a linear constant multivariable system using output feedback is described. The use of this procedure avoids the transformation of variables which is required in other procedures. For the case in which the product of the output and input vector dimensions is greater than the number of states of the plant, we are able to give a rather general solution. In the case in which the states exceed the product of input and output vector dimensions we are able to present a least square solution which may not be stable in all cases. The results are illustrated with examples. 17. Key Words (Suggested by Authoris) (STAR category underlined) optimal control, output feedback Star Category 10 18. Distribution Statement 19. No. of Pages 22. Price's 22. Price's 22. Price's 22. Price's 23. No. of Pages 23. Price's 24. No. of Pages 24. Price's 25. Price's 25. Price's 26. Price's 26. Price's 27. Pric	12. Sponsoring Agancy Name and Address					
16. Abstract The problem of finding a physically realizable, i.e. constant gain, output-feedback controller which will stabilize an unstable plant is one of the oldest and most fundamental problems in control theory. In spite of its long history, this problem still remains unsolved even in the seemingly simple case of linear plants. In this paper, an iterative procedure for determining the constant gain matrix that will stabilize a linear constant multivariable system using output feedback is described. The use of this procedure avoids the transformation of variables which is required in other procedures. For the case in which the product of the output and input vector dimensions is greater than the number of states of the plant, we are able to give a rather general solution. In the case in which the states exceed the product of input and output vector dimensions we are able to present a least square solution which may not be stable in all cases. The results are illustrated with examples. 17. Key Words (Suggested by Author(s)) (STAR estagory underlined) optimal control, output feedback Star Category 10 18. Distribution Statement Optimal control, output feedback Star Category 10 20. Security Clossif, (of this page) 21. No. of Pages 22. Price*	National Aeronautics and Space Administration					
The problem of finding a physically realizable, i.e. constant gain, output- feedback controller which will stabilize an unstable plant is one of the oldest and most fundamental problems in control theory. In spite of its long history, this problem still remains unsolved even in the seemingly simple case of linear plants. In this paper, an iterative procedure for determining the constant gain matrix that will stabilize a linear constant multivariable system using output feedback is described. The use of this procedure avoids the transformation of variables which is required in other procedures. For the case in which the product of the output and input vector dimensions is greater than the number of states of the plant, we are able to give a rather general solution. In the case in which the states exceed the product of input and output vector dimensions we are able to present a least square solution which may not be stable in all cases. The results are illustrated with examples. 17. Key Words (Suggested by Author(s)) (STAR estagory underlined) optimal control, output feedback Star Category 10 18. Distribution Statement optimal control, output feedback Star Category 10 19. Security Clocals, (of this page) 21. No. of Pages 22. Price'				14. Sponsoring Agency	Code	
The problem of finding a physically realizable, i.e. constant gain, output- feedback controller which will stabilize an unstable plant is one of the oldest and most fundamental problems in control theory. In spite of its long history, this problem still remains unsolved even in the seemingly simple case of linear plants. In this paper, an iterative procedure for determining the constant gain matrix that will stabilize a linear constant multivariable system using output feedback is described. The use of this procedure avoids the transformation of variables which is required in other procedures. For the case in which the product of the output and input vector dimensions is greater than the number of states of the plant, we are able to give a rather general solution. In the case in which the states exceed the product of input and output vector dimensions we are able to present a least square solution which may not be stable in all cases. The results are illustrated with examples. 17. Key Words (Suggested by Author(s)) (STAR catagory underlined) Optimal control, output feedback Star Category 10 18. Distribution Statement Optimal control, output feedback Star Category 10 20. Socurity Clouds. (of this page) 21. No. of Pages 22. Price*	15. Supplementary Notes					
The problem of finding a physically realizable, i.e. constant gain, output- feedback controller which will stabilize an unstable plant is one of the oldest and most fundamental problems in control theory. In spite of its long history, this problem still remains unsolved even in the seemingly simple case of linear plants. In this paper, an iterative procedure for determining the constant gain matrix that will stabilize a linear constant multivariable system using output feedback is described. The use of this procedure avoids the transformation of variables which is required in other procedures. For the case in which the product of the output and input vector dimensions is greater than the number of states of the plant, we are able to give a rather general solution. In the case in which the states exceed the product of input and output vector dimensions we are able to present a least square solution which may not be stable in all cases. The results are illustrated with examples. 17. Key Words (Suggested by Author(s)) (STAR catagory underlined) Optimal control, output feedback Star Category 10 18. Distribution Statement Optimal control, output feedback Star Category 10 20. Socurity Clouds. (of this page) 21. No. of Pages 22. Price*						
The problem of finding a physically realizable, i.e. constant gain, output- feedback controller which will stabilize an unstable plant is one of the oldest and most fundamental problems in control theory. In spite of its long history, this problem still remains unsolved even in the seemingly simple case of linear plants. In this paper, an iterative procedure for determining the constant gain matrix that will stabilize a linear constant multivariable system using output feedback is described. The use of this procedure avoids the transformation of variables which is required in other procedures. For the case in which the product of the output and input vector dimensions is greater than the number of states of the plant, we are able to give a rather general solution. In the case in which the states exceed the product of input and output vector dimensions we are able to present a least square solution which may not be stable in all cases. The results are illustrated with examples. 17. Key Words (Suggested by Author(s)) (STAR catagory underlined) Optimal control, output feedback Star Category 10 18. Distribution Statement Optimal control, output feedback Star Category 10 20. Socurity Clouds. (of this page) 21. No. of Pages 22. Price*						
The problem of finding a physically realizable, i.e. constant gain, output- feedback controller which will stabilize an unstable plant is one of the oldest and most fundamental problems in control theory. In spite of its long history, this problem still remains unsolved even in the seemingly simple case of linear plants. In this paper, an iterative procedure for determining the constant gain matrix that will stabilize a linear constant multivariable system using output feedback is described. The use of this procedure avoids the transformation of variables which is required in other procedures. For the case in which the product of the output and input vector dimensions is greater than the number of states of the plant, we are able to give a rather general solution. In the case in which the states exceed the product of input and output vector dimensions we are able to present a least square solution which may not be stable in all cases. The results are illustrated with examples. 17. Key Words (Suggested by Author(s)) (STAR catagory underlined) Optimal control, output feedback Star Category 10 18. Distribution Statement Optimal control, output feedback Star Category 10 20. Socurity Clouds. (of this page) 21. No. of Pages 22. Price*						
feedback controller which will stabilize an unstable plant is one of the oldest and most fundamental problems in control theory. In spite of its long history, this problem still remains unsolved even in the seemingly simple case of linear plants. In this paper, an iterative procedure for determining the constant gain matrix that will stabilize a linear constant multivariable system using output feedback is described. The use of this procedure avoids the transformation of variables which is required in other procedures. For the case in which the product of the output and input vector dimensions is greater than the number of states of the plant, we are able to give a rather general solution. In the case in which the states exceed the product of input and output vector dimensions we are able to present a least square solution which may not be stable in all cases. The results are illustrated with examples. 17. Key Words (Suggested by Author(s)) (STAR estepory underlined) 18. Distribution Statement optimal control, output feedback Star Category 10 19. Security Clessif. (of this report) 20. Security Clessif. (of this page) 21. No. of Pages 22. Price*	16. Abstract					
feedback controller which will stabilize an unstable plant is one of the oldest and most fundamental problems in control theory. In spite of its long history, this problem still remains unsolved even in the seemingly simple case of linear plants. In this paper, an iterative procedure for determining the constant gain matrix that will stabilize a linear constant multivariable system using output feedback is described. The use of this procedure avoids the transformation of variables which is required in other procedures. For the case in which the product of the output and input vector dimensions is greater than the number of states of the plant, we are able to give a rather general solution. In the case in which the states exceed the product of input and output vector dimensions we are able to present a least square solution which may not be stable in all cases. The results are illustrated with examples. 17. Key Words (Suggested by Author(s)) (STAR estepory underlined) 18. Distribution Statement optimal control, output feedback Star Category 10 19. Security Clessif. (of this report) 20. Security Clessif. (of this page) 21. No. of Pages 22. Price*	The problem of finding a physically realizable is a constant asia contant					
and most fundamental problems in control theory. In spite of its long history, this problem still remains unsolved even in the seemingly simple case of linear plants. In this paper, an iterative procedure for determining the constant gain matrix that will stabilize a linear constant multivariable system using output feedback is described. The use of this procedure avoids the transformation of variables which is required in other procedures. For the case in which the product of the output and input vector dimensions is greater than the number of states of the plant, we are able to give a rather general solution. In the case in which the states exceed the product of input and output vector dimensions we are able to present a least square solution which may not be stable in all cases. The results are illustrated with examples. 17. Key Words (Suggested by Authoris) (STAR category underlined) 18. Distribution Statement optimal control, output feedback Star Category 10 18. Security Cleant (of this report) 20. Security Cleant (of this page) 21. No. of Pages 22. Price*						
In this paper, an iterative procedure for determining the constant gain matrix that will stabilize a linear constant multivariable system using output feedback is described. The use of this procedure avoids the transformation of variables which is required in other procedures. For the case in which the product of the output and input vector dimensions is greater than the number of states of the plant, we are able to give a rather general solution. In the case in which the states exceed the product of input and output vector dimensions we are able to present a least square solution which may not be stable in all cases. The results are illustrated with examples. 17. Key Words (Suggested by Author(s)) (STAR category underlined) optimal control, output feedback Star Category 10 18. Distribution Statement 20. Security Clocal. (of this page) 21. No. of Peges 22. Price*						
In this paper, an iterative procedure for determining the constant gain matrix that will stabilize a linear constant multivariable system using output feedback is described. The use of this procedure avoids the transformation of variables which is required in other procedures. For the case in which the product of the output and input vector dimensions is greater than the number of states of the plant, we are able to give a rather general solution. In the case in which the states exceed the product of input and output vector dimensions we are able to present a least square solution which may not be stable in all cases. The results are illustrated with examples. 17. Key Words (Suggested by Author(s)) (STAR estagory underlined) 18. Distribution Statement optimal control, output feedback Star Category 10 19. Security Clease, (of this paper) 21. No. of Pages 22. Price*	this much law still assesses in control theory. In spite of its long history,					
In this paper, an iterative procedure for determining the constant gain matrix that will stabilize a linear constant multivariable system using output feedback is described. The use of this procedure avoids the transformation of variables which is required in other procedures. For the case in which the product of the output and input vector dimensions is greater than the number of states of the plant, we are able to give a rather general solution. In the case in which the states exceed the product of input and output vector dimensions we are able to present a least square solution which may not be stable in all cases. The results are illustrated with examples. 17. Key Words (Suggested by Author(s)) (STAR category underlined) optimal control, output feedback Star Category 10 18. Distribution Statement 19. Security Cleant, (of this report) 20. Security Cleant, (of this page) 21. No. of Pages 22. Price*						
matrix that will stabilize a linear constant multivariable system using output feedback is described. The use of this procedure avoids the transformation of variables which is required in other procedures. For the case in which the product of the output and input vector dimensions is greater than the number of states of the plant, we are able to give a rather general solution. In the case in which the states exceed the product of input and output vector dimensions we are able to present a least square solution which may not be stable in all cases. The results are illustrated with examples. 17. Key Words (Suggested by Author(s)) (STAR category underlined) optimal control, output feedback Star Category 10 18. Distribution Statement 19. Security Clease, (of this report) 20. Security Clease, (of this page) 21. No. of Pages 22. Price*	plants.					
matrix that will stabilize a linear constant multivariable system using output feedback is described. The use of this procedure avoids the transformation of variables which is required in other procedures. For the case in which the product of the output and input vector dimensions is greater than the number of states of the plant, we are able to give a rather general solution. In the case in which the states exceed the product of input and output vector dimensions we are able to present a least square solution which may not be stable in all cases. The results are illustrated with examples. 17. Key Words (Suggested by Author(s)) (STAR category underlined) optimal control, output feedback Star Category 10 18. Distribution Statement 19. Security Clease, (of this report) 20. Security Clease, (of this page) 21. No. of Pages 22. Price*						
feedback is described. The use of this procedure avoids the transformation of variables which is required in other procedures. For the case in which the product of the output and input vector dimensions is greater than the number of states of the plant, we are able to give a rather general solution. In the case in which the states exceed the product of input and output vector dimensions we are able to present a least square solution which may not be stable in all cases. The results are illustrated with examples. 17. Key Words (Suggested by Author(s)) (STAR category underlined) optimal control, output feedback Star Category 10 18. Distribution Statement 19. Security Cleant, (of this report) 20. Security Cleant, (of this page) 21. No. of Pages 22. Price*	In this paper, an iterative procedure for determining the constant gain					
variables which is required in other procedures. For the case in which the product of the output and input vector dimensions is greater than the number of states of the plant, we are able to give a rather general solution. In the case in which the states exceed the product of input and output vector dimensions we are able to present a least square solution which may not be stable in all cases. The results are illustrated with examples. 17. Key Words (Suggested by Author(s)) (STAR category underlined) optimal control, output feedback Star Category 10 19. Security Clease, (of this report) 20. Security Clease, (of this page) 21. No. of Pages 22. Price*	matrix that will stabilize a linear constant multivariable system using output					
variables which is required in other procedures. For the case in which the product of the output and input vector dimensions is greater than the number of states of the plant, we are able to give a rather general solution. In the case in which the states exceed the product of input and output vector dimensions we are able to present a least square solution which may not be stable in all cases. The results are illustrated with examples. 17. Key Words (Suggested by Author(s)) (STAR category underlined) optimal control, output feedback Star Category 10 19. Security Clease, (of this report) 20. Security Clease, (of this page) 21. No. of Pages 22. Price*	feedback is described. The use of this procedure avoids the transformation of					
product of the output and input vector dimensions is greater than the number of states of the plant, we are able to give a rather general solution. In the case in which the states exceed the product of input and output vector dimensions we are able to present a least square solution which may not be stable in all cases. The results are illustrated with examples. 17. Key Words (Suggested by Author(s)) (STAR category underlined) optimal control, output feedback Star Category 10 18. Distribution Statement 21. No. of Pages 22. Price?	variables which is required in other procedures. For the case in which the					
states of the plant, we are able to give a rather general solution. In the case in which the states exceed the product of input and output vector dimensions we are able to present a least square solution which may not be stable in all cases. The results are illustrated with examples. 17. Key Words (Suggested by Author(s)) (STAR category underlined) optimal control, output feedback Star Category 10 18. Distribution Statement 20. Socurity Cleasif. (of this page) 21. No. of Pages 22. Price*	product of the output and input vector dimensions is greater than the number of					
in which the states exceed the product of input and output vector dimensions we are able to present a least square solution which may not be stable in all cases. The results are illustrated with examples. 17. Key Words (Suggested by Author(s)) (STAR category underlined) optimal control, output feedback Star Category 10 18. Distribution Statement optimal control, output feedback Star Category 20 20. Security Classif. (of this page) 21. No. of Pages 22. Price*	states of the plant, we are able to give a mother common colution. To the states					
are able to present a least square solution which may not be stable in all cases. The results are illustrated with examples. 17. Key Words (Suggested by Author(s)) (STAR category underlined) 18. Distribution Statement optimal control, output feedback Star Category 10 19. Security Cleasif. (of this report) 20. Security Cleasif. (of this page) 21. No. of Pages 22. Price*						
The results are illustrated with examples. 17. Key Words (Suggested by Author(s)) (STAR category underlined) 18. Distribution Statement optimal control, output feedback Star Category 10 19. Security Clessif. (of this report) 20. Security Clessif. (of this page) 21. No. of Pages 22. Price*	in which the states exceed the product of input and output vector dimensions we					
17. Key Words (Suggested by Author(s)) (STAR category underlined) 18. Distribution Statement optimal control, output feedback Star Category 10 19. Security Cleasif. (of this report) 20. Security Cleasif. (of this page) 21. No. of Pages 22. Price*						
optimal control, output feedback Star Category 10 19. Security Clessif. (of this report) 20. Security Clessif. (of this page) 21. No. of Pages 22. Price*	The results are illustrated with examples.					
optimal control, output feedback Star Category 10 19. Security Clessif. (of this report) 20. Security Clessif. (of this page) 21. No. of Pages 22. Price*						
optimal control, output feedback Star Category 10 19. Security Clessif. (of this report) 20. Security Clessif. (of this page) 21. No. of Pages 22. Price*						
optimal control, output feedback Star Category 10 19. Security Clessif. (of this report) 20. Security Clessif. (of this page) 21. No. of Pages 22. Price*						
optimal control, output feedback Star Category 10 19. Security Clessif. (of this report) 20. Security Clessif. (of this page) 21. No. of Pages 22. Price*						
optimal control, output feedback Star Category 10 19. Security Clessif. (of this report) 20. Security Clessif. (of this page) 21. No. of Pages 22. Price*						
optimal control, output feedback Star Category 10 19. Security Clessif. (of this report) 20. Security Clessif. (of this page) 21. No. of Pages 22. Price*						
optimal control, output feedback Star Category 10 19. Security Clessif. (of this report) 20. Security Clessif. (of this page) 21. No. of Pages 22. Price*		····				
Star Category 10 19. Security Clessif, (of this report) 20. Socurity Clessif, (of this page) 21. No. of Pages 22. Price*	17. Key Words (Suggested by Author(s)) (STAR catagory underlined) 18. Distribution Statement					
Star Category 10 19. Security Clessif, (of this report) 20. Socurity Clessif, (of this page) 21. No. of Pages 22. Price*	optimal control, output feedback					
19. Security Clessif, (of this report) 20. Socurity Clessif, (of this page) 21. No. of Pages 22. Price						
					,	
	19. Security Classif, (of this report)	20. Security Classif, (of	this page)	21. No. of Perss	22. Price	
	Unclassified	Unclassified		2612	\$3.00	

AN ITERATIVE TECHNIQUE TO STABILIZE A LINEAR TIME INVARIANT MULTIVARIABLE SYSTEM WITH OUTPUT FEEDBACK

Viswanathan Sankaran NRC Postdoctoral Research Associate Flight Dynamics and Control Division NASA Langley Research Center Hampton, Virginia 23665

The problem of finding a physically realizable, i.e. constant gain, output-feedback controller which will stabilize an unstable plant is one of the oldest and most fundamental problems in control theory. In spite of its long history, this problem still remains unsolved even in the seemingly simple case of linear plants.

In this paper, an iterative procedure for determining the constant gain matrix that will stabilize a linear . constant multivariable system using output feedback is described. The use of this procedure avoids the transformation of variables which is required in other procedures. For the case in which the product of the output and input vector dimensions is greater than the number of states of the plant, we are able to give a rather general solution. In the case in which the states exceed the product of input and output vector dimensions we are able to present a least square solution which may not be stable in all cases. The results are illustrated with examples.

1. INTRODUCTION

The design of linear multivariable control systems using output feedback has attracted the attention of many authors [Davison 1970, Fallside and Seraji 1971, Sridhar and Lindorff 1973, Fortmann 1973, Seraji 1973, and others]. There are two ways of approaching this problem. The first method consists of estimating the states of the system using an observer and using these states in the subsequent design. In the second approach, either static or dynamic feedback of the output is used directly in the control problem and this view is adopted here.

Most of the techniques discussed in the literature for approach two require the system matrix to be transformed to a canonical form which may cause some loss of physical insight. This paper describes an iterative procedure for determining the constant gain matrix that will stabilize a linear constant multivariable system using output feedback gains without the necessity of transforming variables while achieving a satisfactory degree of stability and damping ratio. The resultant control problem is algebraic and hence easy to solve.

2. PROBLEM STATEMENT

Consider a linear time-invariant controllable and observable system

$$\dot{x}(t) = A x(t) + Bu(t)$$

$$y(t) = C x(t)$$
(1)

where the state x, input u and output y have dimensions n, m and p, respectively. The output feedback control law is given by

$$u(t) = G y(t)$$
 (2)

where G is a constant mxp feedback gain matrix. Direct substitution of (2) in (1) yields

$$\dot{\mathbf{x}}(t) = (\mathbf{A} + \mathbf{BGC}) \mathbf{x}(t)$$

$$\mathbf{y}(t) = \mathbf{C} \mathbf{x}(t)$$
(3)

The stability of (3) depends primarily on the eigenvalues of A + BGC. If all the eigenvalues have real parts strictly less than $-\nu$ for $\nu > 0$, then the system will be said to have degree of stability ν . The damping ratio of a stable complex eigenvalue is defined as the cosine of its angle with the negative real axis, and is a measure of how oscillatory the trajectory will be. The eigenvalues of A are determined from

$$|A - \lambda_{i}I| = 0, i = 1, 2, \dots, n$$
 (4)

With these basic definitions the next section outlines the procedure for distinct eigenvalues.

PROCEDURE FOR DISTINCT EIGENVALUES

Assume for the time being that the system to be controlled has distinct eigenvalues. If the system matrix A varies to A + &A, then the corresponding variation in the eigenvalue is given by [Rosenbrock 1965]

$$\delta \lambda_{r} = \frac{\text{trace } [Q \cdot \delta A]}{\text{trace } [Q]}$$
 (5)

where

$$Q = \prod_{\substack{i=1\\i\neq r}}^{n} (A - \lambda_{i}I)$$
 (6)

From equation (5) it is possible to write an expression for δA in terms of $\delta \lambda_r$. If we assume that δA results due to the output feedback

of equation (2) with the gain matrix denoted by 8G then

$$\delta A = B \cdot \delta G \cdot C \tag{7}$$

By substituting equation (7) in (5) and simplifying it is possible to write a linear equation for the elements of the gain matrix as

$$\sum_{k=1}^{mp} L_{rk} \left[\delta G_{ij} \right] = \delta \lambda_r \text{ tr } \left[Q \right], r = 1, 2, \dots, n.$$
 (8)

where δG_{ij} is the ijth element of the matrix δG . Since there are n distinct eigenvalues, there are n independent equations. It is thus possible to write the overall equation as

$$P \cdot \delta g = V \tag{9}$$

where P is a known matrix of order n x mp, δg is the mp vector corresponding to the unknown elements of the gain matrix and V is a known constant of order n. Since the equation (5) is only valid for small perturbations, variations in the eigenvalues are applied in small steps in the desired direction to compute δg . An iterative procedure in which the gain elements were computed in small steps to achieve a desired degree of stability and damping ratio was developed and tested on several example problems.

Since the P matrix is of order nxmp, the solution of the unknown mp vector δg depends primarily on the relative magnitude of mp and n. Hence some special cases of interest are treated below:

(i) mp = n: The computation of the gain matrix is unique for a particular problem. In this case P of equation (9) is a square matrix of order n and the computation of the gain matrix is given by

$$\delta g = P^{-1} V \tag{10}$$

(ii) mp > n: In this case (mp - n) elements of the gain matrix may assume arbitrary values and the remaining n elements are computed as above.

(iii) mp < n: In this case we have more equations than the unknowns and the best possible solution is the least square solution of (9) namely, $\delta g = (P^T P)^{-1} P^T V \tag{11}$ However, a stable output feedback is not guaranteed for gain elements computed this way.

A simple necessary condition proposed by Seraji (1973) gives a method to check whether the output feedback gains will stabilize the unstable plant or not.

4. PROCEDURE FOR MULTIPLE EIGENVALUES

The previous section illustrates the procedure for computing the gain matrix for the case in which the eigenvalues are distinct. Difficulty occurs for multiple eigenvalues because equation (5) is then indeterminate. One solution is to differentiate both the numerator and denominator terms of equation (5) with respect to the multiple eigenvalue and use this expression in place of equation (5). The feedback gain perturbations are computed as before. Then using the newly computed gain matrix, the true eigenvalues of the closed loop system matrix are computed. It has been observed that the true eigenvalues often differ from the expected eigenvalues; however, the sum of the changes in the true eigenvalues is nearly equal to the sum of the perturbations requested in the eigenvalues. When this happens, the true eigenvalues are then distinct and the procedure illustrated in the last section may be followed.

5. EXAMPLES

With these theoretical background it is possible to write a computer program. However, because of the complexity of handling complex matrix operations it is not possible to write a general program. Some of the examples are illustrated below:

Example 1: mp = n, distinct eigenvalues. Consider the system given by equation (1) with

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & -4 \end{bmatrix}$$
 (12)

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$
 (13)

$$C = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$
 (14)

Using the program to place the poles to the left of -1 line, the gain matrix using output feedback is given by

$$\mathbf{G} = \begin{bmatrix} 4.02 & 0 \\ -9.03 & 0 \end{bmatrix} \tag{15}$$

and the closed loop system has the following eigenvalues:

Example 2: mp > n, distinct eigenvalues. In the previous example change the B matrix to

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$
 (16)

The gain matrix obtained to place the closed loop poles to the left of -1.0 line then is

$$G = \begin{bmatrix} 4.0378 & 0.0235 \\ -9.047 & -0.0294 \\ 0.043 & 0.0 \end{bmatrix}$$
 (17)

with the last two elements of the third row chosen arbitrarily and the closed loop poles are

Example 3: mp < n, distinct eigenvalues. Consider the examples with

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$
 (18)

$$B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \tag{19}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \tag{20}$$

The open loop poles are at

Using the above procedure the least square solution for the gain matrix being

$$G = \begin{bmatrix} -2.7077 & -4.0921 \end{bmatrix}$$

the closed loop poles are

Note that the least square solution does not stabilize the overall system. As illustrated before, the sign criterion proposed by Seraji fails and hence stabilization by means of output feedback matrix is not possible for this problem.

Example 4: mp = n, multiple eigenvalues. Consider the system

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} -3 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} \mathbf{u}(t) \tag{21}$$

$$y(t) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ & & & \\ 0 & 1 & -1 & 1 \end{bmatrix} x(t)$$
 (22)

The open loop poles has one unstable double pole. Using the procedure given in section 4, the gain matrix obtained being

$$G = \begin{bmatrix} -5.8476 & -4.35 \\ 4.4007 & 2.607 \end{bmatrix}$$
 (23)

and the closed loop poles are

6. CONCLUSIONS

In summary, for a controllable, observable system an iterative technique to determine the constant gain matrix that will stabilize a linear constant multivariable system using output variable feedback is presented. The method consists of adjusting iteratively the unstable eigenvalues in a desired direction to achieve a satisfactory degree of stability and damping ratio. From the development of the procedure it may be seen that the method has the advantage that is not necessary to transform variables and thereby lose physical significance. The resultant control problem also is algebraic and simple to compute.

ACKNOWLEDGEMENTS:

The author wishes to express his gratitude to Mr. Jarrell R. Elliott, Head, Dynamic Analysis Section of NASA Langley Research Center for helpful discussions.

References

Davison, E. J., I.E.E.E., Trans. Automatic Control, 15, 343

Fallside, F., and Seraji, H., 1971, Proceedings I.E.E., 118, 1648

Fortmann, T. E., 1973, Joint Automatic Control Conference, Columbus, 294

Rosenbrock, H. H., 1965, Electronic letter, 1, 278

Seraji, H., 1973, Proceedings I.E.E., 120, 1428

Sridhar, B., and Lindorff, D. P., 1973, Int. J. Control, 18, 993